

Final State Interactions in the Near-Threshold Production of Kaons from Proton-Proton Collisions*

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Abstract

We analyse the $pp \rightarrow p\Lambda K$ cross section recently measured at COSY arguing that the enhancement of the production cross section at energies close to the reaction threshold should be due to the Λp final state interaction. We find that the experimental Λp elastic scattering data as well as the predictions from the Jülich-Bonn model are in reasonable agreement with the new results on K^+ -meson production. We propose to study directly the final state interaction by measurements of the cross section as a function of the hyperon momentum in the Λp cm system.

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Recently, the COSY-11 [1] and COSY-TOF Collaborations [2] measured the $pp \rightarrow p\Lambda K^+$ cross section at energies close to the reaction threshold. The experimental K^+ yield found is twice as large as predicted in Ref. [3] and by Fäldt and Wilkin [4] for excess energies $\epsilon < 7$ MeV whereas both models reproduce the data for $\epsilon > 50$ MeV. Here we will argue that this enhancement is due to Λp Final State Interactions (FSI).

In Ref. [3] the total cross section for the $pp \rightarrow p\Lambda K^+$ reaction has been calculated within the One-Boson-Exchange (OBE) model including both the pion and kaon exchanges in line with the analysis performed at high energies [5]. In the latter work the FSI between the Λ -hyperon and the proton has been neglected for reasons of simplicity but also due to the large uncertainty in the low energy Λp scattering cross section [6, 7]. The free parameters of the model, i.e. the coupling constants and the cut-off parameters of the form factors for the $NN\pi$ and NYK vertices, were fitted to the experimental data taken at high energies [9].

On the other hand, Fäldt and Wilkin [4] calculated the energy dependence of the $pp \rightarrow p\Lambda K^+$ cross section by using only the one pion exchange in line with the calculations from Ref. [8]. Moreover, it was assumed that the production amplitude is constant and is related to the $pp \rightarrow pp\eta$ reaction. The FSI between the Λ -hyperon and the proton then was incorporated via the effective range approximation. With only a single free parameter fixed by the η production data, the calculations from Ref. [4] reasonably reproduced the $pp \rightarrow p\Lambda K^+$ cross section at $\epsilon = 2$ MeV [10] available at that time. Note, however, that the FSI correction factor at this energy is about $\simeq 14$ and therefore the production amplitude due to the pion exchange itself is very small. On the other hand, Tsushima et al. [11] calculated the $pp \rightarrow p\Lambda K^+$ production amplitude in a microscopic model which illustrates that near threshold the pion exchange contribution to the reaction is small and underestimates the most recent data from COSY [1, 2] by about a factor $\simeq 10$. Our present work is to clarify these partly conflicting results.

We recall that the total cross section for the reaction $pp \rightarrow p\Lambda K^+$ is obtained by integrating the differential cross section $d^2\sigma/dtds_1$ over the available phase space,

$$\sigma = \int dtds_1 \frac{d^2\sigma}{dtds_1} = \frac{1}{2^9\pi^3q^2s} \int dtds_1 \frac{q_K}{\sqrt{s_1}} |M(t, s_1)|^2. \quad (1)$$

Here s is the squared invariant mass of the colliding protons, q is the proton momentum in the center-of-mass while t is the squared 4-momentum transferred from the initial proton to the final hyperon or proton in case of the kaon or pion exchange, respectively. Moreover, s_1 is the squared invariant mass of the Kp or $K\Lambda$ system, respectively, while q_K is the kaon momentum in the corresponding center-of-mass system. In Eq. (1) $|M|$ is the amplitude of the reaction which is an analytical function of t and s_1 .

Let us start with the experimental $pp \rightarrow p\Lambda K^+$ cross section and extract the reaction amplitude averaged over t and s_1 by taking $|M|^2$ out of the integral in Eq. (1). The average reaction amplitude then can be determined by comparing σ from Eq. (1) with the corresponding experimental cross section from Refs. [1, 2, 9]. The results for the average matrix element $|M|$ are shown in Fig. 1 as function of

the excess energy $\epsilon = \sqrt{s} - m_p - m_\Lambda - m_K$, with m_p , m_Λ and m_K being the mass of the proton, Λ -hyperon and kaon, respectively. Obviously, the matrix element $|M|$ as evaluated from the data is not constant and decreases substantially with the excess energy ϵ .

Following the Watson-Migdal approximation the total reaction amplitude can be factorized in terms of the production $|M_{prod}|$ and FSI amplitude $|A_{FSI}|$. Since at high energies the FSI is negligible, the latter amplitude should converge to 1 for $\epsilon \rightarrow \infty$. Within the OBE model the squared production amplitude, for instance for the pion exchange, is given as [3, 5, 8]¹

$$|M_{prod}(t, s_1)|^2 = g_{NN\pi}^2 \frac{t}{(t - \mu^2)^2} \left[\frac{\Lambda_\pi^2 - \mu^2}{\Lambda_\pi^2 - t} \right]^2 |A_{\pi^0 p \rightarrow \Lambda K^+}(s_1)|^2, \quad (2)$$

where $g_{NN\pi}$ is the coupling constant and μ is the pion mass. The form factor of the $NN\pi$ vertex is given in brackets with Λ_π denoting the cut-off parameter. In Eq. (2) $|A|$ is the amplitude for the reaction $\pi^0 p \rightarrow \Lambda K^+$, which can be calculated microscopically within the resonance model² or evaluated from the experimental data as

$$|A_{\pi^0 p \rightarrow \Lambda K^+}(s_1)|^2 = 16\pi s_1 \frac{q_\pi}{q_K} \sigma_{\pi^0 p \rightarrow \Lambda K^+}(s_1), \quad (3)$$

where q_π is the pion momentum in the ΛK center-of-mass system and $\sigma_{\pi^0 p \rightarrow \Lambda K^+}$ is the physical cross section.

Since $m_\Lambda + m_K \leq \sqrt{s_1} \leq m_\Lambda + m_K + \epsilon$, thus close to the $pp \rightarrow p\Lambda K^+$ reaction threshold the amplitude $|A_{\pi^0 p \rightarrow \Lambda K^+}|$ is almost constant. Moreover, the 4-momentum transfer squared t is a slowly varying function of energy at low ϵ as shown in Fig. 2a). Therefore it is a quite reasonable approximation to assume that the production amplitude $|M_{prod}|$ is almost constant near the threshold. Similar arguments can be set up for the kaon exchange amplitude. We thus conclude that the deviation of the reaction amplitude $|M|$ (shown in Fig. 1) from a constant value is due to the FSI.

A similar conclusion is obtained by analysing the $pp \rightarrow pp\eta$ reaction cross section [12] in the same way as illustrated in Fig. 3. Moreover, the experimental data indicate that for kaon production the FSI correction is substantially smaller than for η -meson production. We note that in principle one should account for the FSI between all final particles produced in the reaction $pp \rightarrow p\Lambda K^+$; here we assume that the Λp interaction is much stronger than the $K^+ p$ and $K^+ \Lambda$ interaction, respectively.

Following the original idea of Chew and Low [13] the amplitude $|A_{FSI}|$ is related to the on mass-shell elastic scattering amplitude $|A_{el}|$ for the reaction $\Lambda p \rightarrow \Lambda p$, which can be calculated from the corresponding physical cross section in analogy to Eq. (3). The strength of the FSI depends upon the Λ -hyperon momentum in the Λp center-of-mass system and for fixed excess energy the relative momentum q_Λ extends from zero to its maximal value as shown in Fig. 2b). Thus to calculate the total cross section for the reaction $pp \rightarrow p\Lambda K^+$ at $\epsilon < 100$ MeV [1, 2] one needs to know the Λp scattering amplitude for $0 \leq q_\Lambda \leq 400$ MeV/c.

¹One should add the amplitude corresponding to the exchange graph.

²Since the resonance properties are fitted to the experimental data, both the resonance model and Eq. (3) should give the same results.

Fig. 4 shows the amplitude $|A_{el}|$ extracted from the experimental data [9] plotted as a function of q_Λ . Actually, there are no data below 50 MeV/c and the experimental results have large error bars. The amplitude $|A_{el}|$, furthermore, can be calculated in the hyperon-nucleon interaction model developed by Holzenkamp, Reuber, Holinde and Speth [7]. Within the Bonn-Jülich approach the Λp elastic scattering cross section is given in the effective-range formalism as

$$\sigma_{\Lambda p \rightarrow \Lambda p} = \frac{\pi}{q_\Lambda^2 + (-1/a_s + 0.5r_s q_\Lambda^2)^2} + \frac{3\pi}{q_\Lambda^2 + (-1/a_t + 0.5r_t q_\Lambda^2)^2}, \quad (4)$$

where a is the scattering length and r is the effective range, while the indices s and t stand for singlet and triplet Λp states. The scattering amplitude $|A_{el}|$ was calculated with a and r parameters from Ref. [7] (model \tilde{A}) and is shown in Fig. 4 by the dashed line. Actually the effective range approximation is valid at low energies and can not be extended to $q_\Lambda > 200$ MeV/c. Following both the low energy prediction from the Bonn-Jülich model and the experimental data we can fit the Λp scattering amplitude as

$$|A_{el}| = C \left[1 + \frac{\alpha\beta}{q_\Lambda^2 + \alpha^2/4} \right] \quad (5)$$

with $\alpha = 170$ MeV, $\beta = 130$ MeV and $C = 87$. The result is shown by the fat solid line in Fig. 4.

The FSI amplitude now is proportional to the Λp elastic scattering amplitude, but normalized such that $|A_{FSI}| \rightarrow 1$ at large excess energies ϵ . The solid line in Fig. 1 shows our result calculated with the $|A_{FSI}|$ averaged over the phase space distribution for q_Λ and the production amplitude $|M_{prod}| = 1.05$ fm. The dashed line in Fig. 1 illustrates the result obtained with the prescription for FSI from Ref. [4, 14] as

$$|A_{FSI}|^2 = \frac{2\beta_s^2}{(\alpha_s + \sqrt{\alpha_s^2 + 2\mu\epsilon})^2} + \frac{4\beta_t^2}{(\alpha_t + \sqrt{\alpha_t^2 + 2\mu\epsilon})^2}, \quad (6)$$

where μ is the reduced mass in the Λp system and the parameters α and β were evaluated from the scattering length and the effective range for the singlet and spin-triplet Λp interaction [7]. Fig. 1 illustrates a good agreement between the experimental data from COSY [1, 2] and our FSI approach and proves the strong influence of the Λp interaction in the final state.

Actually, the FSI effect can be observed directly by measuring the $pp \rightarrow p\Lambda K^+$ cross section as a function of the momentum q_Λ . Fig. 5 shows this differential cross section calculated³ by the phase space alone (dashed lines) and with FSI correction (solid lines) for the excess energies ϵ relevant for COSY-11 and COSY-TOF experiments. The enhancement at low q_Λ is due to the FSI and is more pronounced for the range of TOF energies.

We conclude, that the $pp \rightarrow p\Lambda K^+$ cross section at energies close to the reaction threshold is strongly effected by the hyperon-nucleon final state interaction. However, this effect is less pronounced as in the $pp \rightarrow pp\eta$ reaction close to threshold since the ΛN interaction is weaker than the pp interaction at low relative momenta.

³The results are normalized to the experimental total cross section.

Further experimental studies are necessary for the direct measurement of the FSI as a function of the relative momentum q_Λ as well as more upgrade calculations on the Yp interaction, which are under study in Jülich [15].

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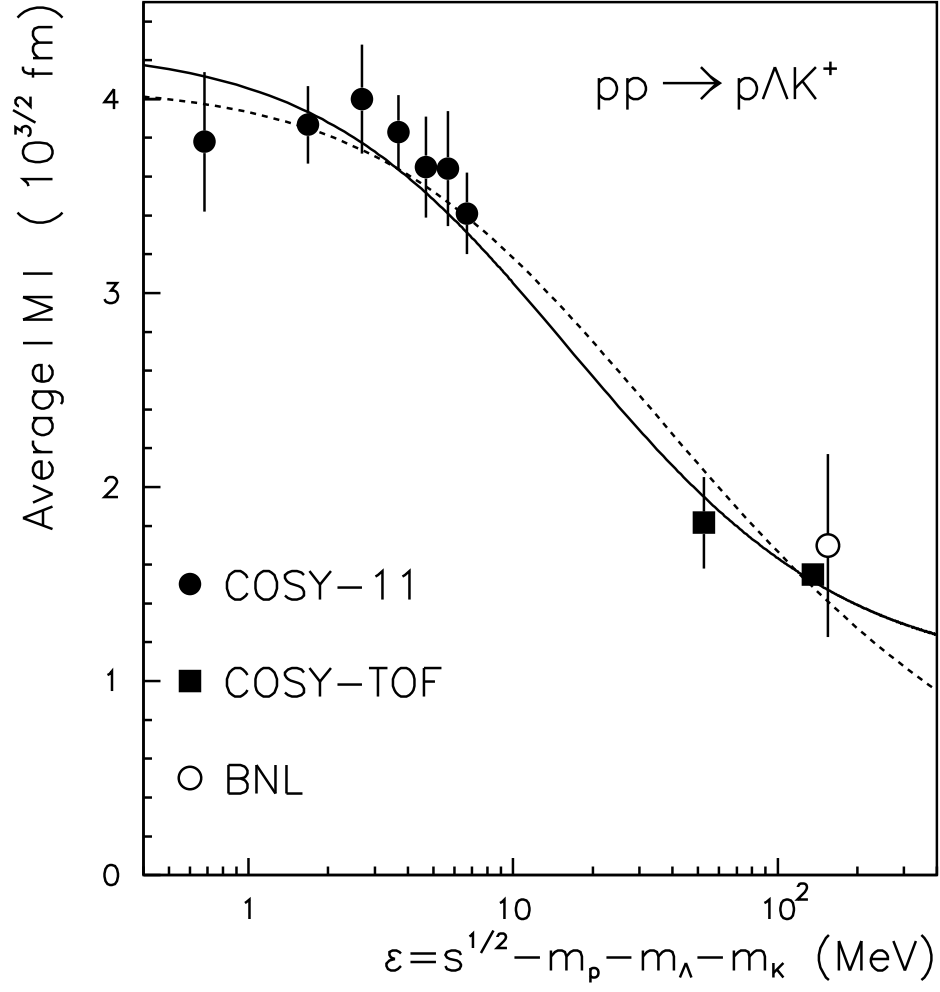


Figure 1: The average amplitude for the $pp \rightarrow p\Lambda K^+$ reaction as a function of the excess energy. The symbols show the results extracted from the experimental data of Refs. [1, 2, 9]. The lines are our calculations with FSI as described in the text.

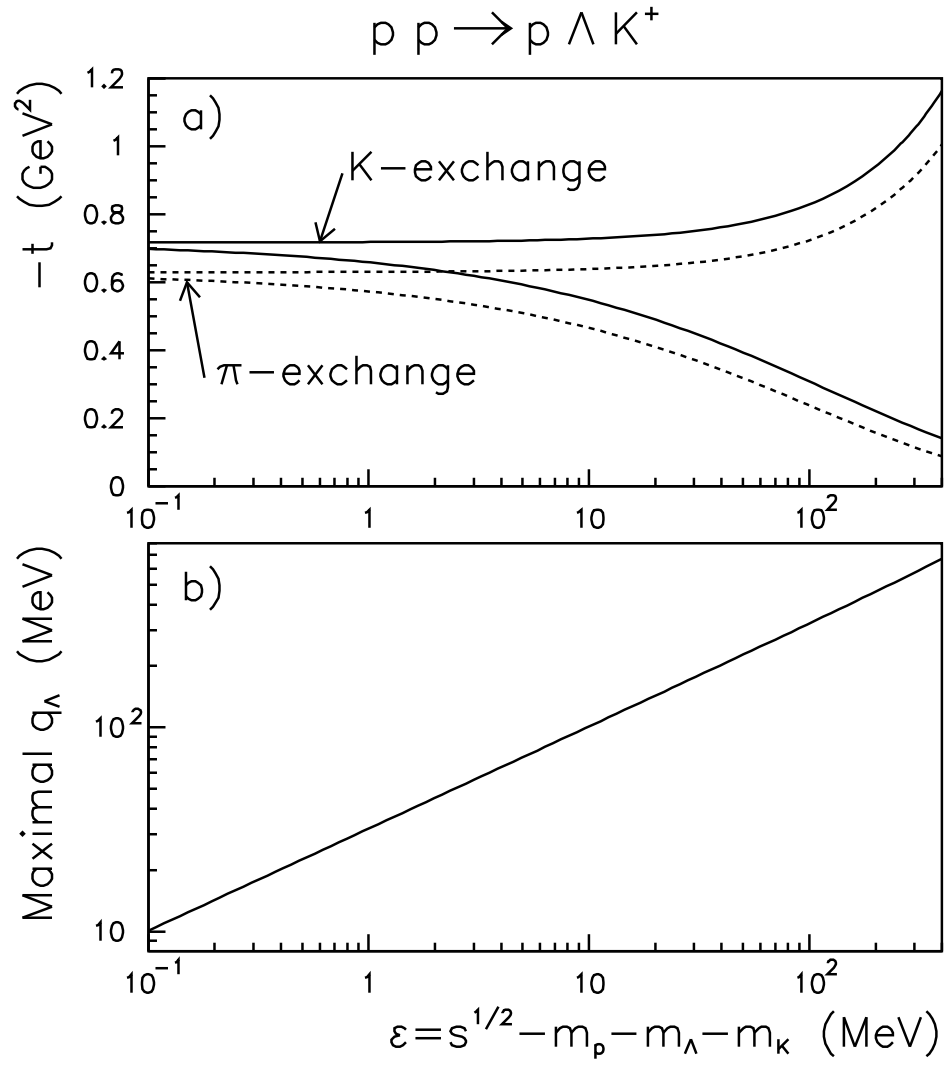


Figure 2: The range of the 4-momentum transfer squared (a) and maximal hyperon momentum in the Λp cm system (b) as function of the excess energy ϵ .

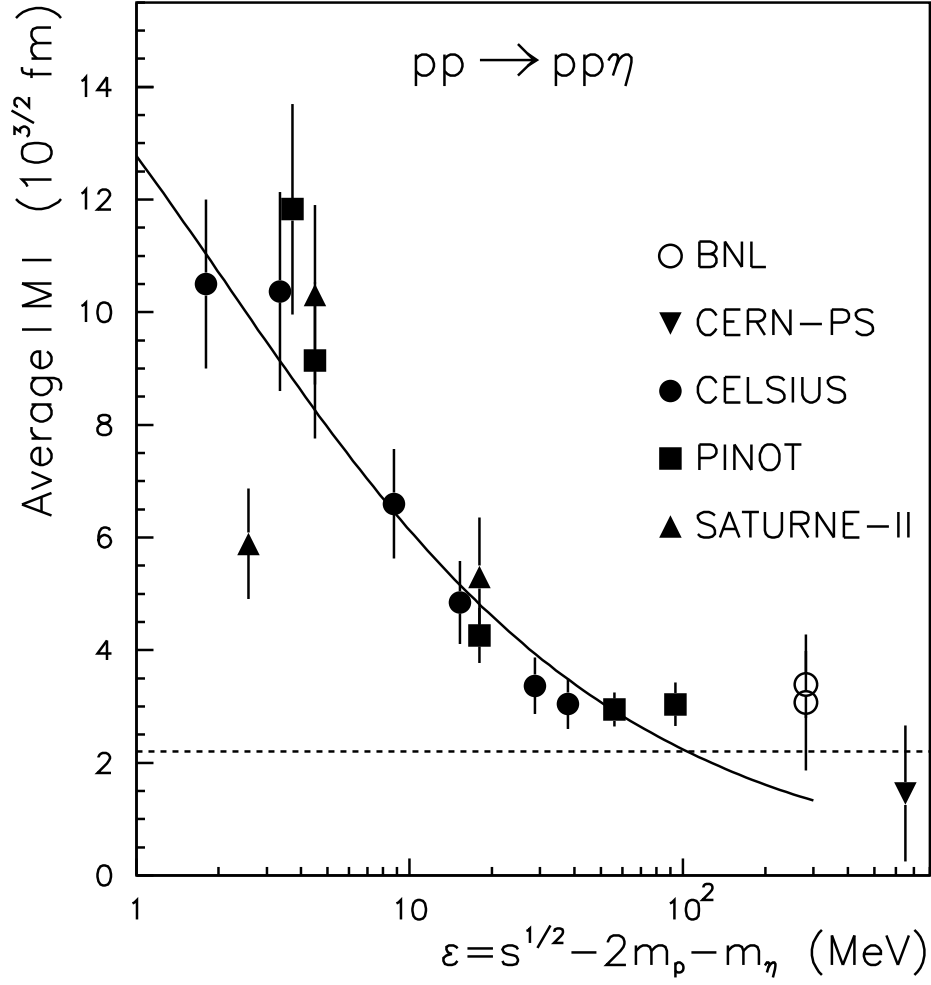


Figure 3: The average amplitude for the $pp \rightarrow pp\eta$ reaction. The symbols show the results extracted from the experimental data [12]. The dashed line shows the production amplitude while the solid line is the total amplitude corrected by FSI (6) with parameters in line with the pp interaction.

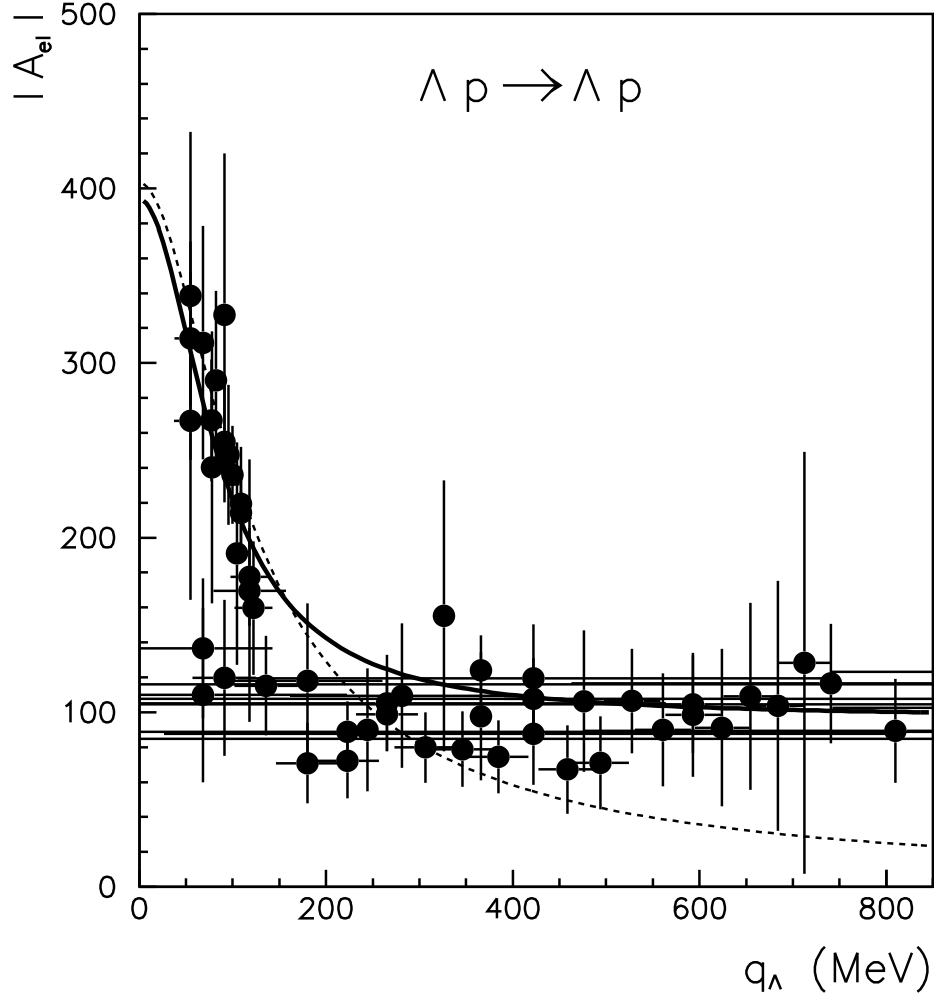


Figure 4: The Λp elastic scattering amplitude as a function of the hyperon momentum q_Λ in the center-of-mass system. The dots show the experimental data [9]; the fat solid line is our fit while the dashed line shows the effective range approximation with parameters from the Jülich-Bonn model [7].

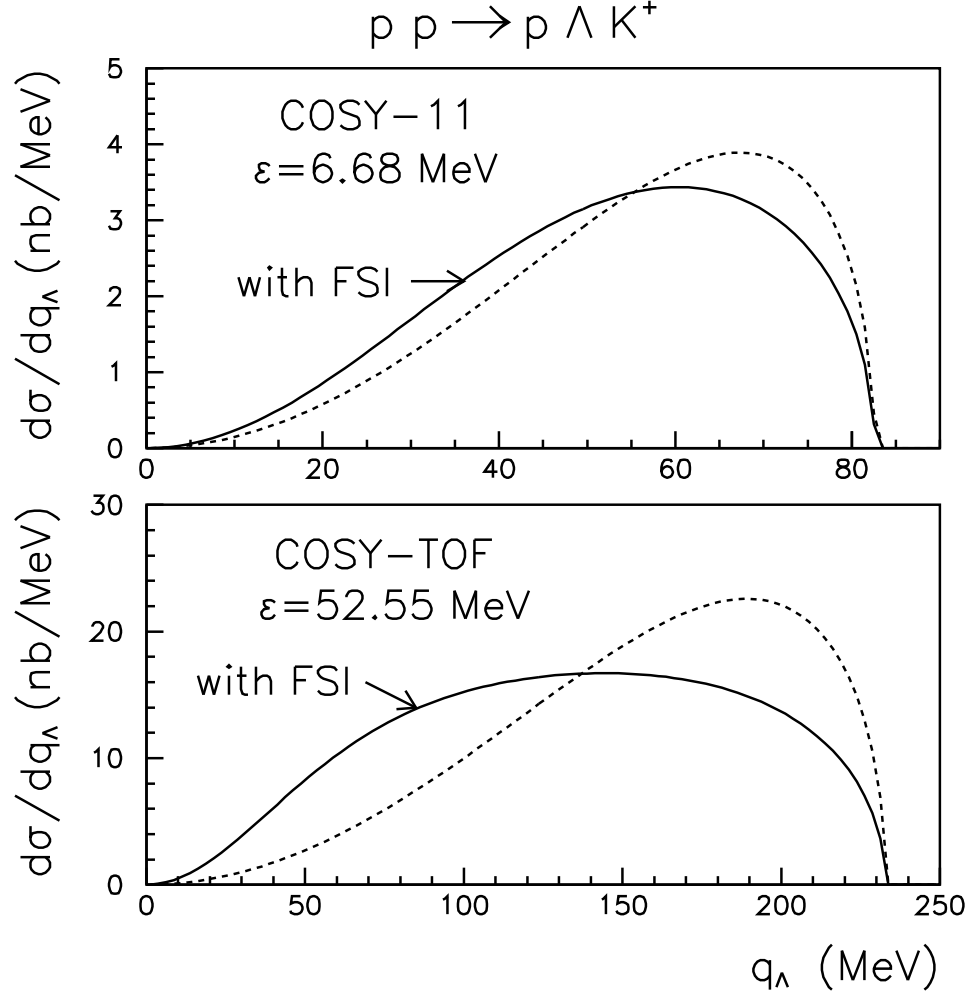


Figure 5: The $pp \rightarrow p\Lambda K^+$ cross section as a function of the hyperon momentum in the Λp cm system calculated for two values of the excess energy ϵ . The solid lines show our results with FSI correction while the dashed lines are the pure phase space distributions.